NOVAE THEORIAE
FUNCTIONUM SYMMETRARUM
SPECIMEN.

SCRIPSIT

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TURICI,

TYPIS ET SUMPTIBUS FRIDERICI SCHULTHESSII.

1 8 3 7.

AD AUDIENDAM ORATIONEM IN ADEUNDO MUNERE MATHESEOS PROFESSORIS ORDINARII IN ACADEMIA TURICENSI

DIE IV. NOVEMBRIS MDCCCXXXVII

HABENDAM

OBSERVANTER ATQUE HUMANISSIME

INVITAT

ANTONIUS MUELLER.

TURICI,

Typis et sumptibus Friderici Schulthessii.

1 8 3 7.

Quindecimviri scholis rei publicae Turicensis regundis maxime colendi benevole me vocarunt, qui provinciam disciplinarum mathematicarum in Academia Turicensi docendarum suscipiam. In adeundo hoc ipso munere ut id faciam, quod et veteri consuetudine receptum est et legibus praescriptum, die IV. Novembris h. a. orationem publice sum habiturus; quo die ut benigne me audiant, his pagellis omnes literarum cultores et fautores summa reverentia et humanitate invito.

Sed quum in academiis bene ordinatis nihil agatur, nisi quod ad amplificandam cognitionem quid conferat, hac mihi data occasione novae functionum symmetrarum theoriae specimen cum mathematicis communicandum statui.

Per trecentos enim fere annos omnium quidem sed nefaria virium contentione ac summo studio mathematici id egerunt, ut directam altiorum graduum aequationes solvendi methodum inveniant. Nam postquam *Eulerus* anno 1732 certam radicum formam divinatione quadam invenit, et viginti annis post in novam omnes omnium graduum aequationes solvendi methodum incidit, quae huic analyseos parti antea obductae sunt tenebrae, dispulsae quidem visae sunt, et spes inde affulsit, fore ut problema difficillimum et obstinatissimum sane solvi tandem possit. Et eodem fere tempore, quo *Eulerus*, etiam *Bezoltius* (Bezout) explananda solvendi methodo generali quadam in eam opinionem rem adduxit, ut summam voti eamque omnium virorum doctorum mox consequi liceat. Deinde viginti annis post omnia, quae usque tum ad illustrandam quaestionem publicata fuerunt, *Lagrangius* percensuit, qui ea disquisitione in novam adductus est viam, qua recto secureque ad finem pervenire posse visus est; sed inchoatam duntaxat quaestionem posteris reliquit. Hoc ipsum *Lagrangii* opus, uti omnium, quae antea de aequationibus algebraice solvendis scripta sunt, plane optimum est, ita etiam fines per hos sexaginta et septem annos superiores haudquaquam transgressos constituit. Nam licet ex illo tempore naviter strenueque analystae animum huic rei adverterent, tamen nemo ad felicem eam eventum pervenisse contendat, nisi vagas quasdam veras esse aut demonstratione munitas opiniones putet. Fuere enim, tum qui se magnum illud solvendi rationem problema invenisse crediderint, donec se in errorem incidisse cernentes, id omnino non posse solvi asseveraverint; tum qui et eam propositionem simul demonstrare voluerint: algebraice aequationum, in altioribus quidem earum gradibus radices haud posse explicari; tum denique, qui hujus problematis solutionem inveniri posse non prorsus quidem negaverint, sed tanta opus esse contenderint calculi algebraici, quem ipsa solutio requirat, mole et prolixitate, ut hanc posse superari omnino desperaverint.

Quod quum igitur gravissimum illud neque summo labore neque omni studio vinci potuit problema, aliam sibi rei partem tractandam mathematici sumserunt. Namque eam proposuerunt quaestionem: quo modo aequationum, quarum coefficientes numeri sint dati, radices si non exacte, per quandam tamen approximationem inveniri possint. Eique quaestioni inde a *Vietae* temporibus, qui ingeniosam radices aequationum per approximationem extrahendi methodum jam dedit, usque ad hunc diem quam plurimi mathematici summa industria incumbebant, et nostris quoque temporibus prae ceteris analystae student. At vero ne nunc quidem tantae tamque variae contentiones ad id certe perduxisse videntur, ut postulato illi lenissimo sane satisfieri posse putes.

Jam vero si omnes colligas, quos in aequationes earumque solutionem analystae scripserunt commentarios, certamque inde consequi studeas sententiam de magni illius problematis solutione; quaerenti: num ea omnibus partibus absoluta, et rite doctrinae praeceptis conveniens dari possit, duae praecipue, de quibus haud facile dubites, observationes in conspectu erunt. Namque id primum luce clarius mihi esse videtur, prorsus ab eo mathematicos aberrasse, quod natura et vera analyseos indoles postulet, dum ad extractionem radicum per approximationem confugiunt. Deinde vero etiam negari nequit, nihil sane ex iis, quae hucusque de aequationibus algebraice solvendis scripta sunt, hauriri et pro argumento haberi posse, quo nisus aequationes altiorum graduum non esse solvendas recte asseveres. Nam ea certe argumenta, quibus analystae quidam ad demonstrandam propositionem, omnes aequationes altiorum graduum omnino solvi non posse usi sunt, propterea nulla sunt, quod permultae eaeque ipsae altiorum graduum aequationes particulares ab *Moivraeo*, *Eulero* et aliis algebraice revera solutae sunt, neque tamen ex iis ipsis argumentis, cur harum duntaxat aequationum radices algebraice exprimi possint, eluceat. Sed ii, qui solutionem per se quidem fieri, verumtamen nimiam, quam postulat, calculi et molem et prolixitatem superari non posse censent, nihil aliud, quam magnum illud problema et acuminis et assiduitatis et voluntatis constantiae plus requirere quam ipsi habeant, aperte confiteri videntur.

Quam ob rem qui matheseos naturam et indolem haud prorsus ignorant et aspernantur, neque illud certe dubitabunt, quin aequationum earumque omnium ordinum radices per coefficientes exacte exprimi possint. Atqui in eo analystae hucusque erraverunt, quod opinati sunt, ad gravissimum illud problema recte tractandum pauca sufficere et parva ea subsidia, quorum usus jam patuit. Quantula enim necessariorum fortasse subsidiorum sit copia, accurate perpensa eorum, quibus uti licet, conditione apud omnes manifestum fit. Nec quisquam nescit, quanti radicum functiones symmetrae in tractanda illa quaestione sint momenti; sed reputa, quaeso, et probe considera, quid ad illustrandas illas functiones analystae revera perfecerint. Enim vero nonnullas quidem relationes jam diu erutas habuerunt, quibus adjuvantibus symmetrae illius functionis, cujus exponentes sunt numeri dati, valor potest computari; sed numquid haec subsidia expedita illa et generalia sunt, quae analyseos ratio et non impeditus requirit ingressus? Verum vero asymmetrae etiam functiones, quas dicunt, in auxilium vocandae sunt; sed tamen ad eas disquirendas nunc quoque analystae vix viam aggressi sunt.

Quare si igitur vera sunt omnia, quae modo dixi, jam quoque concedas necesse est, immensum manare et novarum et ante omnia tractandarum quaestionum campum, nec ullam solvendi gravissimum illud problema spem esse relictam, sed omnem hujus rei memoriam deponi oportere, donec omnes cujuscunque generis et conditionis radicum pariter ac coefficientium functiones, quae huc spectant, eo usque sint explicatae atque expeditae, ut simplicium elementorum instar in analyticis operationibus iis uti possis.

Quapropter jam id agam, ut ad expediendas pro doctrinae indole symmetras illas functiones rectam viam justamque rationem proponam.

1.

Sint

$$x_1 \quad x_2 \quad x_3 \quad \ldots\ldots \quad x_n \tag{1}$$

quantitates diversae, quas aut radices aequationis n^{ti} ordinis, aut aliis quibuscunque affectionibus praeditas sumere liceat.

Deinde per signum

$$\mathfrak{S}\,(x_1^a \;\; x_2^b \;\; x_3^c \;\ldots\; x_n^z) \tag{2}$$

summa indicetur omnium inde oriundorum productorum, quod omnibus, quibus modis possunt, transponantur in producto

$$x_1^a \;\; x_2^b \;\; x_3^c \;\ldots\ldots\; x_n^z$$

exponentes a b c .... z, iique immutata elementorum $x_1 \; x_2 \; x_3 \ldots x_n$ serie.

Quarum vero summarum casus simplicissimi ut breviter exprimantur, in universum ponatur esse

$$\mathfrak{S}\,(x_1^1 \;\; x_2^1 \;\; x_3^1 \;\ldots\; x_p^1 \;\; x_{p+1}^0 \;\ldots\; x_n^0) = A_p \tag{3}$$

Quibus sumtionibus praemissis, jam sequens problema solvendum est:

»Exprimere quamque functionem $\mathfrak{S}\,(x_1^a \;\; x_2^b \;\ldots\; x_n^z)$ per quantitates $A_1 \; A_2 \; A_3 \ldots\ldots$«

Sed in solvendo eo problemate viam primum indicabo, qua leges eruendae sint, e quibus majorum exponentium functiones $\mathfrak{S}\,(\ldots.)$ ad minorum exponentium functiones $\mathfrak{S}\,(\ldots.)$ reduci queant; deinde quibus sub conditionibus functionum $\mathfrak{S}\,(\ldots.)$ valores per quantitatum $A_1 \; A_2 \; A_3 \ldots.$ functiones expressos colligere liceat, demonstrabo.

2.

Cujus autem disquisitionis exordium a justa eorum calculorum incipiat expeditione, qui per signa

$$\mathfrak{S}\,(x_1^a \;\; x_2^0 \;\ldots\ldots\ldots\; x_n^0).\; A_p$$
$$\mathfrak{S}\,(x_1^a \;\; x_2^b \;\; x_3^0 \;\ldots\ldots\; x_n^0).\; A_p$$
$$\mathfrak{S}\,(x_1^a \;\; x_2^b \;\; x_3^c \;\; x_4^0 \;\ldots\; x_n^0).\; A_p$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$

indicati sunt, quorum producta idoneo ordine signisque supra positis convenienter digerantur. Qua quidem ratione ea, quae sequntur, theoremata prodeunt:

$$\begin{aligned}\mathfrak{S}\,(x_1^a \; x_2^0 \ldots x_n^0).\, A_p &= \mathfrak{S}\,(x_1^{a+1} \; x_2^1 \ldots\ldots x_p^1 \; x_{p+1}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^1 \ldots\ldots x_{p+1}^1 \; x_{p+2}^0 \ldots x_n^0)\end{aligned} \tag{4}$$

$$\begin{aligned}\mathfrak{S}\,(x_1^a \; x_2^b \; x_3^0 \ldots x_n^0).\, A_p &= \mathfrak{S}\,(x_1^{a+1} \; x_2^{b+1} \; x_3^1 \ldots x_p^1 \; x_{p+1}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^{a+1} \; x_2^b \; x_3^1 \ldots x_{p+1}^1 \; x_{p+2}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^{b+1} \; x_3^1 \ldots x_{p+1}^1 \; x_{p+2}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^b \; x_3^1 \ldots x_{p+2}^1 \; x_{p+3}^0 \ldots x_n^0)\end{aligned} \tag{5}$$

$$\begin{aligned}\mathfrak{S}\,(x_1^a \; x_2^b \; x_3^c \; x_4^0 \ldots x_n^0).\, A_p &= \mathfrak{S}\,(x_1^{a+1} \; x_2^{b+1} \; x_3^{c+1} \; x_4^1 \ldots x_p^1 \; x_{p+1}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^{a+1} \; x_2^{b+1} \; x_3^c \; x_4^1 \ldots x_{p+1}^1 \; x_{p+2}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^{a+1} \; x_2^b \; x_3^{c+1} \; x_4^1 \ldots x_{p+1}^1 \; x_{p+2}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^{b+1} \; x_3^{c+1} \; x_4^1 \ldots x_{p+1}^1 \; x_{p+2}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^{a+1} \; x_2^b \; x_3^c \; x_4^1 \ldots x_{p+2}^1 \; x_{p+3}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^{b+1} \; x_3^c \; x_4^1 \ldots x_{p+2}^1 \; x_{p+3}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^b \; x_3^{c+1} \; x_4^1 \ldots x_{p+2}^1 \; x_{p+3}^0 \ldots x_n^0)\\ &+ \mathfrak{S}\,(x_1^a \; x_2^b \; x_3^c \; x_4^1 \ldots x_{p+3}^1 \; x_{p+4}^0 \ldots x_n^0)\end{aligned} \tag{6}$$

$$\left.\begin{aligned}
\mathfrak{C}(x_1^a\, x_2^b\, x_3^c\, x_4^d\, x_5^o \ldots x_n^o).\,A_p = \;& \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{d+1}\, x_5^1 \ldots x_p^1\, x_{p+1}^o \ldots\ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^d\, x_5^1 \ldots\ldots x_{p+1}^1\, x_{p+2}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^c\, x_4^{d+1}\, x_5^1 \ldots\ldots x_{p+1}^1\, x_{p+2}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^b\, x_3^{c+1}\, x_4^{d+1}\, x_5^1 \ldots\ldots x_{p+1}^1\, x_{p+2}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^{b+1}\, x_3^{c+1}\, x_4^{d+1}\, x_5^1 \ldots\ldots x_{p+1}^1\, x_{p+2}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^c\, x_4^d\, x_5^1 \ldots\ldots\ldots x_{p+2}^1\, x_{p+3}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^b\, x_3^{c+1}\, x_4^d\, x_5^1 \ldots\ldots\ldots x_{p+2}^1\, x_{p+3}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^b\, x_3^c\, x_4^{d+1}\, x_5^1 \ldots\ldots\ldots x_{p+2}^1\, x_{p+3}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^{b+1}\, x_3^{c+1}\, x_4^d\, x_5^1 \ldots\ldots\ldots x_{p+2}^1\, x_{p+3}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^{b+1}\, x_3^c\, x_4^{d+1}\, x_5^1 \ldots\ldots\ldots x_{p+2}^1\, x_{p+3}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^b\, x_3^{c+1}\, x_4^{d+1}\, x_5^1 \ldots\ldots\ldots x_{p+2}^1\, x_{p+3}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^{a+1}\, x_2^b\, x_3^c\, x_4^d\, x_5^1 \ldots\ldots\ldots\ldots x_{p+3}^1\, x_{p+4}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^{b+1}\, x_3^c\, x_4^d\, x_5^1 \ldots\ldots\ldots\ldots x_{p+3}^1\, x_{p+4}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^b\, x_3^{c+1}\, x_4^d\, x_5^1 \ldots\ldots\ldots\ldots x_{p+3}^1\, x_{p+4}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^b\, x_3^c\, x_4^{d+1}\, x_5^1 \ldots\ldots\ldots\ldots x_{p+3}^1\, x_{p+4}^o \ldots x_n^o)\\
+\;& \mathfrak{C}(x_1^a\, x_2^b\, x_3^c\, x_4^d\, x_5^1 \ldots\ldots\ldots\ldots\ldots x_{p+4}^1\, x_{p+5}^o \ldots x_n^o)
\end{aligned}\right\}\quad(7)$$

Quae quum simplicem progressionis legem eamque constantem satis luculenter exhibeant, generali termino haud opus est.

3.

In propositione (4) autem primum ponatur $a - h$ loco ipsius a, et $p + h$ loco ipsius p; deinde termini singuli per $(-1)^h$ multiplicentur; denique ipsi h tribuantur valores $0, 1, 2, 3 \ldots a-1$, et quae hac ratione nascuntur aequationes, colligantur in unam; quae quidem breviter sic exhibetur:

$$\begin{aligned}
\Sigma_h\, (-1)^h.\, \mathfrak{C}(x_1^{a-h}\, x_2^o \ldots x_n^o).\, A_{p+h} = \;& \Sigma_h\, (-1)^h.\, \mathfrak{C}(x_1^{a-h+1}\, x_2^1 \ldots x_{p+h}^1\, x_{p+h+1}^o \ldots x_n^o)\\
+\;& \Sigma_h\, (-1)^h.\, \mathfrak{C}(x_1^{a-h}\, x_2^1 \ldots x_{p+h+1}^1\, x_{p+h+2}^o \ldots x_n^o)\\
& h = o, 1, 2, \ldots\ldots a-1.
\end{aligned}$$

Jam vero est

$$\begin{aligned}
\Sigma_h\, (-1)^h.\, \mathfrak{C}(x_1^{a-h+1}\, x_2^1 \ldots x_{p+h}^1\, x_{p+h+1}^o \ldots x_n^o) &=\\
h = o, 1, 2, \ldots a-1 \qquad &\\
&= \mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_p^1\, x_{p+1}^o \ldots x_n^o)\\
&- \Sigma_m\, (-1)^m.\, \mathfrak{C}(x_1^{a-m}\, x_2^1 \ldots x_{p+m+1}^1\, x_{p+m+2}^o \ldots x_n^o)\\
&\qquad m = o, 1, 2, \ldots a-2
\end{aligned}$$

et

$$\begin{aligned}
\Sigma_h\, (-1)^h.\, \mathfrak{C}(x_1^{a-h}\, x_2^1 \ldots x_{p+h+1}^1\, x_{p+h+2}^o \ldots x_n^o) &=\\
h = o, 1, 2, \ldots a-1 \qquad &\\
&= \Sigma_m\, (-1)^m.\, \mathfrak{C}(x_1^{a-m}\, x_2^1 \ldots x_{p+m+1}^1\, x_{p+m+2}^o \ldots x_n^o)\\
&+ (-1)^{a-1}.\, (p+a).\, \mathfrak{C}(x_1^1\, x_2^1 \ldots x_{p+a}^1\, x_{p+a+1}^o \ldots x_n^o)\\
&\qquad m = o, 1, 2, \ldots a-2
\end{aligned}$$

ergo

$$\begin{aligned}
\Sigma_h\, (-1)^h.\, \mathfrak{C}(x_1^{a-h}\, x_2^o \ldots x_p^o).\, A_{p+h} &= \mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_p^1\, x_{p+1}^o \ldots x_n^o)\\
h = o, 1, 2, \ldots a-1 \qquad &\\
&+ (-1)^{a-1}.\, (p + a).\, \mathfrak{C}(x_1^1 \ldots x_{p+a}^1\, x_{p+a+1}^o \ldots x_n^o)
\end{aligned}$$

Hinc sequitur theorema:

$$\left.\begin{aligned}\mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) &= (-1)^a.\,(p+a).\,A_{p+a}\\ &+ \Sigma_h\,(-1)^h.\,\mathfrak{C}(x_1^{a-h}\, x_2^0 \ldots x_n^0).\,A_{p+h}\\ &\qquad h = o, 1, 2, \ldots\ldots a-1\end{aligned}\right\}\quad(8)$$

Quodsi autem in generali hac propositione ipsi p valorem 1 tribuis, theorema illud *Neutoni* satis notum prodit:

$$\left.\begin{aligned}\mathfrak{C}(x_1^{a+1}\, x_2^0 \ldots\ldots x_n^0) &= (-1)^a.\,(a+1).\,A_{a+1}\\ &+ \Sigma_h\,(-1)^h.\,\mathfrak{C}(x_1^{a-h}\, x_2^0 \ldots x_n^0).\,A_{h+1}\\ &\qquad h = o, 1, 2, \ldots a-1\end{aligned}\right\}\quad(9)$$

4.

In propositione (5) vero primum ponatur $b-i$ loco ipsius b, et $p+i$ loco ipsius p; deinde singuli termini per $(-1)^i$ multiplicentur; tum ipsi i tribuantur valores $0, 1, 2, \ldots\ldots b-1$, et aequationes denique hinc ortae in unam colligantur. Quibus rite expeditis, cum sit

$$\begin{aligned}&\Sigma_i\,(-1)^i.\,\mathfrak{C}(x_1^{a+1}\, x_2^{b-i+1}\, x_3^1 \ldots x_{p+i}^1\, x_{p+i+1}^0 \ldots\ldots x_n^0) =\\ &\qquad i = 0, 1, 2, \ldots\ldots b-1\\ &\quad = \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^1 \ldots\ldots x_p^1\, x_{p+1}^0 \ldots x_n^0)\\ &\quad - \Sigma_m\,(-1)^m.\,\mathfrak{C}(x_1^{a+1}\, x_2^{b-m}\, x_3^1 \ldots x_{p+m+1}^1\, x_{p+m+2}^0 \ldots x_n^0)\\ &\qquad m = o, 1, 2, \ldots b-2\end{aligned}$$

$$\begin{aligned}&\Sigma_i\,(-1)^i.\,\mathfrak{C}(x_1^{a+1}\, x_2^{b-i}\, x_3^1 \ldots\ldots x_{p+i+1}^1\, x_{p+i+2}^0 \ldots\ldots x_n^0) =\\ &\qquad i = o, 1, 2, \ldots b-1\\ &\quad = \Sigma_m\,(-1)^m.\,\mathfrak{C}(x_1^{a+1}\, x_2^{b-m}\, x_3^1 \ldots x_{p+m+1}^1\, x_{p+m+2}^0 \ldots\ldots x_n^0)\\ &\quad + (-1)^{b-1}.\,(p+b-1).\,\mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_{p+b}^1\, x_{p+b+1}^0 \ldots\ldots x_n^0)\\ &\qquad m = o, 1, 2, \ldots\ldots b-2\end{aligned}$$

$$\begin{aligned}&\Sigma_i\,(-1)^i.\,\mathfrak{C}(x_1^{a}\, x_2^{b-i+1}\, x_3^1 \ldots x_{p+i+1}^1\, x_{p+i+2}^0 \ldots x_n^0) =\\ &\qquad i = o, 1, 2, \ldots b-1\\ &\quad = \mathfrak{C}(x_1^{a}\, x_2^{b+1}\, x_3^1 \ldots\ldots x_{p+1}^1\, x_{p+2}^0 \ldots\ldots x_n^0)\\ &\quad - \Sigma_m\,(-1)^m.\,\mathfrak{C}(x_1^{a}\, x_2^{b-m}\, x_3^1 \ldots\ldots x_{p+m+2}^1\, x_{p+m+3}^0 \ldots x_n^0)\\ &\qquad m = o, 1, 2, \ldots\ldots b-2\end{aligned}$$

$$\begin{aligned}&\Sigma_i\,(-1)^i.\,\mathfrak{C}(x_1^{a}\, x_2^{b-i}\, x_3^1 \ldots x_{p+i+2}^1\, x_{p+i+3}^0 \ldots x_n^0) =\\ &\qquad i = o, 1, 2, \ldots b-1\\ &\quad = \Sigma_m\,(-1)^m.\,\mathfrak{C}(x_1^{a}\, x_2^{b-m}\, x_3^1 \ldots x_{p+m+2}^1\, x_{p+m+3}^0 \ldots x_n^0)\\ &\quad + (-1)^{b-1}.\,(p+b).\,\mathfrak{C}(x_1^{a}\, x_2^1 \ldots x_{p+b+1}^1\, x_{p+b+2}^0 \ldots x_n^0)\\ &\qquad m = o, 1, 2, \ldots\ldots b-2\end{aligned}$$

simplex haec procedit aequatio:

$$\left.\begin{aligned}&\Sigma_i\,(-1)^i.\,\mathfrak{C}(x_1^{a}\, x_2^{b-i}\, x_3^0 \ldots x_n^0).\,A_{p+i} =\\ &\qquad i = o, 1, 2, \ldots b-1\\ &\quad = \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^1 \ldots x_p^1\, x_{p+1}^0 \ldots\ldots x_n^0)\\ &\quad + \mathfrak{C}(x_1^{a}\, x_2^{b+1}\, x_3^1 \ldots x_{p+1}^1\, x_{p+2}^0 \ldots\ldots x_n^0)\\ &\quad + (-1)^{b-1}.\,(p+b-1).\,\mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_{p+b}^1\, x_{p+b+1}^0 \ldots\ldots x_n^0)\\ &\quad + (-1)^{b-1}.\,(p+b).\,\mathfrak{C}(x_1^{a}\, x_2^1 \ldots x_{p+b+1}^1\, x_{p+b+2}^0 \ldots x_n^0)\end{aligned}\right\}\quad(\pi)$$

Eaque ipsa in aequatione (π) primum ponatur a — h loco ipsius a, et p + h loco ipsius p; deinde singuli per $(-1)^h$ multiplicentur termini; tum ipsi h valores 0, 1, 2, ... a — 1 tribuantur, quo facto in unam denique colligantur aequationes ex ea ratione prodeuntes. Jam vero cum sit

$$\Sigma_h\ (-1)^h.\ \mathfrak{C}(x_1^{a-h+1}\ x_2^{b+1}\ x_3^1 \dots x_{p+h}^1\ x_{p+h+1}^0 \dots x_n^0) =$$
$$h = 0, 1, 2, \dots a-1$$
$$= \mathfrak{C}(x_1^{a+1}\ x_2^{b+1}\ x_3^1 \dots x_p^1\ x_{p+1}^0 \dots x_n^0)$$
$$- \Sigma_r\ (-1)^r.\ \mathfrak{C}(x_1^{a-r}\ x_2^{b+1}\ x_3^1 \dots x_{p+r+1}^1\ x_{p+r+2}^0 \dots x_n^0)$$
$$r = 0, 1, 2, \dots a-2$$

$$\Sigma_h\ (-1)^h.\ \mathfrak{C}(x_1^{a-h}\ x_2^{b+1}\ x_3^1 \dots x_{p+h+1}^1\ x_{p+h+2}^0 \dots x_n^0) =$$
$$h = 0, 1, 2, \dots a-1$$
$$= \Sigma_r\ (-1)^r.\ \mathfrak{C}(x_1^{a-r}\ x_2^{b+1}\ x_3^1 \dots x_{p+r+1}^1\ x_{p+r+2}^0 \dots x_n^0)$$
$$+ (-1)^{a-1}.\ (p+a-1).\ \mathfrak{C}(x_1^{b+1}\ x_2^1 \dots x_{p+a}^1\ x_{p+a+1}^0 \dots x_n^0)$$
$$r = 0, 1, 2, \dots a-2$$

$$\Sigma_h\ (-1)^h.\ (p+h+b-1).\ \mathfrak{C}(x_1^{a-h+1}\ x_2^1 \dots x_{p+h+b}^1\ x_{p+h+b+1}^0 \dots x_n^0) =$$
$$h = 0, 1, 2, \dots a-1$$
$$= (p+b-1).\ \mathfrak{C}(x_1^{a+1}\ x_2^1 \dots x_{p+b}^1\ x_{p+b+1}^0 \dots x_n^0)$$
$$- \Sigma_r\ (-1)^r.\ (p+r+b).\ \mathfrak{C}(x_1^{a-r}\ x_2^1 \dots x_{p+r+b+1}^1\ x_{p+r+b+2}^0 \dots x_n^0)$$
$$r = 0, 1, 2, \dots a-2$$

$$\Sigma_h\ (-1)^h.\ (p+h+b).\ \mathfrak{C}(x_1^{a-h}\ x_2^1 \dots x_{p+h+b+1}^1\ x_{p+h+b+2}^0 \dots x_n^0) =$$
$$h = 0, 1, 2, \dots a-1$$
$$= \Sigma_r\ (-1)^r.\ (p+r+b).\ \mathfrak{C}(x_1^{a-r}\ x_2^1 \dots x_{p+r+b+1}^1\ x_{p+r+b+2}^0 \dots x_n^0)$$
$$+ (-1)^{a-1}.\ (p+a+b-1).\ (p+a+b).\ \mathfrak{C}(x_1^1 \dots x_{p+a+b}^1\ x_{p+a+b+1}^0 \dots x_n^0)$$
$$r = 0, 1, 2, \dots a-2$$

oriatur inde necesse est aequatio:

$$\Sigma_h\ (-1)^h.\ \Sigma_i\ (-1)^i.\ \mathfrak{C}(x_1^{a-h}\ x_2^{b-i}\ x_3^0 \dots x_n^0).\ A_{p+h+i} =$$
$$h = 0, 1, 2, \dots a-1$$
$$i = 0, 1, 2, \dots b-1$$
$$= \mathfrak{C}(x_1^{a+1}\ x_2^{b+1}\ x_3^1 \dots x_p^1\ x_{p+1}^0 \dots x_n^0)$$
$$+ (-1)^{a-1}.\ (p+a-1).\ \mathfrak{C}(x_1^{b+1}\ x_2^1 \dots x_{p+a}^1\ x_{p+a+1}^0 \dots x_n^0)$$
$$+ (-1)^{b-1}.\ (p+b-1).\ \mathfrak{C}(x_1^{a+1}\ x_2^1 \dots x_{p+b}^1\ x_{p+b+1}^0 \dots x_n^0)$$
$$+ (-1)^{a+b-2}.\ (p+a+b).\ (p+a+b-1).\ \mathfrak{C}(x_1^1 \dots x_{p+a+b}^1\ x_{p+a+b+1}^0 \dots x_n^0)$$

Quare concludere licet, esse

$$\left.\begin{aligned}
&\mathfrak{C}(x_1^{a+1}\ x_2^{b+1}\ x_3^1 \dots x_p^1\ x_{p+1}^0 \dots x_n^0) = \\
&\quad = (-1)^{a+b-1}.\ (p+a+b)^{2/-1}.\ A_{p+a+b} \\
&\quad + (-1)^b.\ (p+b-1).\ \mathfrak{C}(x_1^{a+1}\ x_2^1 \dots x_{p+b}^1\ x_{p+b+1}^0 \dots x_n^0) \\
&\quad + (-1)^a.\ (p+a-1).\ \mathfrak{C}(x_1^{b+1}\ x_2^1 \dots x_{p+a}^1\ x_{p+a+1}^0 \dots x_n^0) \\
&\quad + \Sigma_h\ (-1)^h.\ \Sigma_i\ (-1)^i.\ \mathfrak{C}(x_1^{a-h}\ x_2^{b-i}\ x_3^0 \dots x_n^0).\ A_{p+h+i} \\
&\qquad h = 0, 1, 2, \dots a-1 \\
&\qquad i = 0, 1, 2, \dots b-1
\end{aligned}\right\}\ (10)$$

et casu particulari, quo p = 2,

$$\left.\begin{aligned}
&\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{o} \ldots x_n^{o}) = \\
&\qquad = (-1)^{a+b-1}.\ (a+b+2)^{2/-1}.\ A_{a+b+2} \\
&\qquad + (-1)^{b}.\ (b+1).\ \mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_{b+2}^{1}\, x_{b+3}^{o} \ldots x_n^{o}) \\
&\qquad + (-1)^{a}.\ (a+1).\ \mathfrak{C}(x_1^{b+1}\, x_2^{1} \ldots x_{a+2}^{1}\, x_{a+3}^{o} \ldots x_n^{o}) \\
&\qquad + \Sigma_h\, (-1)^{h}.\ \Sigma_i\, (-1)^{i}.\ \mathfrak{C}(x_1^{a-h}\, x_2^{b-i}\, x_3^{o} \ldots x_n^{o}).\ A_{h+i+2} \\
&\qquad\qquad\qquad h = o, 1, 2, \ldots a-1 \\
&\qquad\qquad\qquad i = o, 1, 2, \ldots b-1
\end{aligned}\right\} \quad (11)$$

5.

Jam vero in propositione (6) primum ponatur c — k loco ipsius c, et p + k loco ipsius p; deinde singuli per $(-1)^k$ multiplicentur termini; denique ipsi k valores tribuantur 0, 1, 2, ... c — 1, et quae inde proveniunt aequationes, in unam colligantur. Quibus rite peractis, hanc veram esse elucebit relationem:

$$\left.\begin{aligned}
&\Sigma_k\, (-1)^{k}.\ \mathfrak{C}(x_1^{a}\, x_2^{b}\, x_3^{c-k}\, x_4^{o} \ldots x_n^{o}).\ A_{p+k} = \\
&\qquad k = o, 1, 2, \ldots\ldots c-1 \\
&= \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{1} \ldots x_p^{1}\, x_{p+1}^{o} \ldots\ldots x_n^{o}) \\
&+ \mathfrak{C}(x_1^{a+1}\, x_2^{b}\, x_3^{c+1}\, x_4^{1} \ldots x_{p+1}^{1}\, x_{p+2}^{o} \ldots\ldots x_n^{o}) \\
&+ \mathfrak{C}(x_1^{a}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{1} \ldots x_{p+1}^{1}\, x_{p+2}^{o} \ldots\ldots x_n^{o}) \\
&+ \mathfrak{C}(x_1^{a}\, x_2^{b}\, x_3^{c+1}\, x_4^{1} \ldots\ldots x_{p+2}^{1}\, x_{p+3}^{o} \ldots\ldots x_n^{o}) \\
&+ (-1)^{c-1}.\ (p+c-2).\ \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{1} \ldots x_{p+c}^{1}\, x_{p+c+1}^{o} \ldots x_n^{o}) \\
&+ (-1)^{c-1}.\ (p+c-1).\ \mathfrak{C}(x_1^{a+1}\, x_2^{b}\, x_3^{1} \ldots x_{p+c+1}^{1}\, x_{p+c+2}^{o} \ldots x_n^{o}) \\
&+ (-1)^{c-1}.\ (p+c-1).\ \mathfrak{C}(x_1^{a}\, x_2^{b+1}\, x_3^{1} \ldots x_{p+c+1}^{1}\, x_{p+c+2}^{o} \ldots x_n^{o}) \\
&+ (-1)^{c-1}.\ (p+c).\qquad \mathfrak{C}(x_1^{a}\, x_2^{b}\, x_3^{1} \ldots\ldots x_{p+c+2}^{1}\, x_{p+c+3}^{o} \ldots x_n^{o})
\end{aligned}\right\} \quad (\pi)$$

Qua tamen in aequatione (π) primum ponatur b — i loco ipsius b, et p + i loco ipsius p; deinde singuli per $(-1)^i$ multiplicentur termini; valores 0, 1, 2, 3 ..... b — 1 denique ipsi i tribuantur, inque unam aequationes ita ortae colligantur. Quo facto hanc veram esse videbis relationem:

$$\left.\begin{aligned}
&\Sigma_i\, (-1)^{i}.\ \Sigma_k\, (-1)^{k}.\ \mathfrak{C}(x_1^{a}\, x_2^{b-i}\, x_3^{c-k}\, x_4^{o} \ldots\ldots x_n^{o}).\ A_{p+i+k} = \\
&\qquad i = 0, 1, 2, \ldots b-1;\ k = 0, 1, 2, \ldots\ldots c-1 \\
&= \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{1} \ldots x_p^{1}\, x_{p+1}^{o} \ldots\ldots x_n^{o}) \\
&+ \mathfrak{C}(x_1^{a}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{1} \ldots x_{p+1}^{1}\, x_{p+2}^{o} \ldots\ldots x_n^{o}) \\
&+ (-1)^{b-1}.\ (p+b-2).\ \mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{p+b}^{1}\, x_{p+b+1}^{o} \ldots x_n^{o}) \\
&+ (-1)^{b-1}.\ (p+b-1).\ \mathfrak{C}(x_1^{a}\, x_2^{c+1}\, x_3^{1} \ldots x_{p+b+1}^{1}\, x_{p+b+2}^{o} \ldots x_n^{o}) \\
&+ (-1)^{c-1}.\ (p+c-2).\ \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{1} \ldots x_{p+c}^{1}\, x_{p+c+1}^{o} \ldots x_n^{o}) \\
&+ (-1)^{c-1}.\ (p+c-1).\ \mathfrak{C}(x_1^{a}\, x_2^{b+1}\, x_3^{1} \ldots x_{p+c+1}^{1}\, x_{p+c+2}^{o} \ldots x_n^{o}) \\
&+ (-1)^{b+c-2}.\ (p+b+c-1)^{2/-1}.\ \mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_{p+b+c}^{1}\, x_{p+b+c+1}^{o} \ldots x_n^{o}) \\
&+ (-1)^{b+c-2}.\ (p+b+c)^{2/-1}.\ \mathfrak{C}(x_1^{a}\, x_2^{1} \ldots x_{p+b+c+1}^{1}\, x_{p+b+c+2}^{o} \ldots x_n^{o})
\end{aligned}\right\} \quad (\pi^1)$$

Sed in hac iterum aequatione (π^1) primum ponatur a — h loco ipsius a, et p + h loco ipsius p; quibus positis singuli termini per $(-1)^h$ multiplicentur; et tributis denique ipsi h valoribus 0, 1, 2, ... a — 1, in unam quae inde nascuntur aequationes colligantur. Qua igitur ratione haec prodit relatio:

$$\Sigma_h (-1)^h . \Sigma_i (-1)^i . \Sigma_k (-1)^k . \mathfrak{C}(x_1^{a-h}\, x_2^{b-i}\, x_3^{c-k}\, x_4^{o} \ldots x_n^{o}) . A_{p+h+i+k} =$$
$$h = 0, 1, 2, \ldots a-1;\ i = 0, 1, 2, \ldots b-1;\ k = 0, 1, 2, \ldots c-1$$
$$\begin{aligned}
&= \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{1} \ldots x_p^{1}\, x_{p+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a-1} . (p + a - 2) . \mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{p+a}^{1}\, x_{p+a+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{b-1} . (p + b - 2) . \mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{p+b}^{1}\, x_{p+b+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{c-1} . (p + c - 2) . \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{1} \ldots x_{p+c}^{1}\, x_{p+c+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+b-2} . (p + a + b - 1)^{2/-1} . \mathfrak{C}(x_1^{c+1}\, x_2^{1} \ldots x_{p+a+b}^{1}\, x_{p+a+b+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+c-2} . (p + a + c - 1)^{2/-1} . \mathfrak{C}(x_1^{b+1}\, x_2^{1} \ldots x_{p+a+c}^{1}\, x_{p+a+c+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{b+c-2} . (p + b + c - 1)^{2/-1} . \mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_{p+b+c}^{1}\, x_{p+b+c+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+b+c-3} . (p + a + b + c)^{3/-1} . \mathfrak{C}(x_1^{1} \ldots x_{p+a+b+c}^{1}\, x_{p+a+b+c+1}^{o} \ldots x_n^{o})
\end{aligned}$$

Quae si autem vera est, veram quoque hanc esse concludere licet:

$$\left.\begin{aligned}
&\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{1} \ldots x_p^{1}\, x_{p+1}^{o} \ldots x_n^{o}) =\\
&= (-1)^{a+b+c-2} . (p + a + b + c)^{3/-1} . A_{p+a+b+c}\\
&+ (-1)^{b+c-1} . (p+b+c-1)^{2/-1} . \mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_{p+b+c}^{1}\, x_{p+b+c+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+c-1} . (p+a+c-1)^{2/-1} . \mathfrak{C}(x_1^{b+1}\, x_2^{1} \ldots x_{p+a+c}^{1}\, x_{p+a+c+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+b-1} . (p+a+b-1)^{2/-1} . \mathfrak{C}(x_1^{c+1}\, x_2^{1} \ldots x_{p+a+b}^{1}\, x_{p+a+b+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{c} . (p+c-2) . \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{1} \ldots x_{p+c}^{1}\, x_{p+c+1}^{o} \ldots x^{n}_{o})\\
&+ (-1)^{b} . (p+b-2) . \mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{p+b}^{1}\, x_{p+b+1}^{o} \ldots x_n^{o})\\
&+ (-1)^{a} . (p+a-2) . \mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{p+a}^{1}\, x_{p+a+1}^{o} \ldots x_n^{o})\\
&+ \Sigma_h (-1)^h . \Sigma_i (-1)^i . \Sigma_k (-1)^k . \mathfrak{C}(x_1^{a-h}\, x_2^{b-i}\, x_3^{c-k}\, x_4^{o} \ldots x_n^{o}) . A_{p+h+i+k}\\
&\qquad h = 0, 1, 2, 3, \ldots a-1\\
&\qquad i = 0, 1, 2, 3, \ldots b-1\\
&\qquad k = 0, 1, 2, 3, \ldots c-1
\end{aligned}\right\} \quad (12)$$

et particulari casu, quo $p = 3$:

$$\left.\begin{aligned}
&\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{o} \ldots x_n^{o}) =\\
&= (-1)^{a+b+c-2} . (a + b + c + 3)^{3/-1} . A_{a+b+c+3}\\
&+ (-1)^{b+c-1} . (b + c + 2)^{2/-1}\ \mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_{b+c+3}^{1}\, x_{b+c+4}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+c-1} . (a + c + 2)^{2/-1}\ \mathfrak{C}(x_1^{b+1}\, x_2^{1} \ldots x_{a+c+3}^{1}\, x_{a+c+4}^{o} \ldots x_n^{o})\\
&+ (-1)^{a+b-1} . (a + b + 2)^{2/-1}\ \mathfrak{C}(x_1^{c+1}\, x_2^{1} \ldots x_{a+b+3}^{1}\, x_{a+b+4}^{o} \ldots x_n^{o})\\
&+ (-1)^{c} . (c + 1) . \mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{1} \ldots x_{c+3}^{1}\, x_{c+4}^{o} \ldots x_n^{o})\\
&+ (-1)^{b} . (b + 1) . \mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{b+3}^{1}\, x_{b+4}^{o} \ldots x_n^{o})\\
&+ (-1)^{a} . (a + 1) . \mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^{1} \ldots x_{a+3}^{1}\, x_{a+4}^{o} \ldots x_n^{o})\\
&+ \Sigma_h (-1)^h . \Sigma_i (-1)^i . \Sigma_k (-1)^k . \mathfrak{C}(x_1^{a-h}\, x_2^{b-i}\, x_3^{c-k}\, x_4^{o} \ldots x_n^{o}) . A_{h+i+k+3}\\
&\qquad h = 0, 1, 2, \ldots a-1\\
&\qquad i = 0, 1, 2, \ldots b-1\\
&\qquad k = 0, 1, 2, \ldots c-1
\end{aligned}\right\} \quad (13)$$

6.

Sed et inde, quod relatio (7) sit vera, iisdem fere calculis adhibitis, sequentem quoque veram esse concludere licet relationem:

$$
\left.\begin{array}{l}
\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{d+1}\, x_5^1 \ldots\ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) = \\
= (-1)^{a+b+c+d-3}.\,(p+a+b+c+d)^{4/-1}.\; A_{p+a+b+c+d} \\
+ (-1)^{b+c+d-2}.\,(p+b+c+d-1)^{3/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^1 . x_{p+b+c+d}^1\, x_{p+b+c+d+1}^0 . x_n^0) \\
+ (-1)^{a+c+d-2}.\,(p+a+c+d-1)^{3/-1}.\,\mathfrak{C}(x_1^{b+1}\, x_2^1 . x_{p+a+c+d}^1\, x_{p+a+c+d+1}^0 . x_n^0) \\
+ (-1)^{a+b+d-2}.\,(p+a+b+d-1)^{3/-1}.\,\mathfrak{C}(x_1^{c+1}\, x_2^1 . x_{p+a+b+d}^1\, x_{p+a+b+d+1}^0 . x_n^0) \\
+ (-1)^{a+b+c-2}.\,(p+a+b+c-1)^{3/-1}.\,\mathfrak{C}(x_1^{d+1}\, x_2^1 . x_{p+a+b+c}^1\, x_{p+a+b+c+1}^0 . x_n^0) \\
+ (-1)^{c+d-1}.\,(p+c+d-2)^{2/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^1 \ldots x_{p+c+d}^1\, x_{p+c+d+1}^0 \ldots x_n^0) \\
+ (-1)^{b+d-1}.\,(p+b+d-2)^{2/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^1 \ldots x_{p+b+d}^1\, x_{p+b+d+1}^0 \ldots x_n^0) \\
+ (-1)^{b+c-1}.\,(p+b+c-2)^{2/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^{d+1}\, x_3^1 \ldots x_{p+b+c}^1\, x_{p+b+c+1}^0 \ldots x_n^0) \\
+ (-1)^{a+d-1}.\,(p+a+d-2)^{2/-1}.\,\mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^1 \ldots x_{p+a+d}^1\, x_{p+a+d+1}^0 \ldots x_n^0) \\
+ (-1)^{a+c-1}.\,(p+a+c-2)^{2/-1}.\,\mathfrak{C}(x_1^{b+1}\, x_2^{d+1}\, x_3^1 \ldots x_{p+a+c}^1\, x_{p+a+c+1}^0 \ldots x_n^0) \\
+ (-1)^{a+b-1}.\,(p+a+b-2)^{2/-1}.\,\mathfrak{C}(x_1^{c+1}\, x_2^{d+1}\, x_3^1 \ldots x_{p+a+b}^1\, x_{p+a+b+1}^0 \ldots x_n^0) \\
+ (-1)^{d}.\,(p+d-3).\,\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^1 \ldots x_{p+d}^1\, x_{p+d+1}^0 \ldots x_n^0) \\
+ (-1)^{c}.\,(p+c-3).\,\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{d+1}\, x_4^1 \ldots x_{p+c}^1\, x_{p+c+1}^0 \ldots x_n^0) \\
+ (-1)^{b}.\,(p+b-3).\,\mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^{d+1}\, x_4^1 \ldots x_{p+b}^1\, x_{p+b+1}^0 \ldots x_n^0) \\
+ (-1)^{a}.\,(p+a-3).\,\mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^{d+1}\, x_4^1 \ldots x_{p+a}^1\, x_{p+a+1}^0 \ldots x_n^0) \\
+ \Sigma_h (-1)^h.\, \Sigma_i (-1)^i.\, \Sigma_k (-1)^k.\, \Sigma_l (-1)^l.\, \mathfrak{C}(x_1^{a-h}\, x_2^{b-i}\, x_3^{c-k}\, x_4^{d-l}\, x_5^0 \ldots x_n^0).\, A_{p+h+i+k+l} \\
\quad h = 0, 1, 2, \ldots a-1;\; i = 0, 1, 2, \ldots b-1;\; k = 0, 1, 2, \ldots c-1;\; l = 0, 1, 2 \ldots d-1
\end{array}\right\} \quad (14)
$$

Quaeque si iterum vera sit, hanc etiam veram esse oportet:

$$
\left.\begin{array}{l}
\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^{d+1}\, x_5^0 \ldots\ldots x_n^0) = \\
= (-1)^{a+b+c+d-3}.\,(a+b+c+d+4)^{4/-1}.\; A_{a+b+c+d+4} \\
+ (-1)^{b+c+d-2}.\,(b+c+d+3)^{3/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_{b+c+d+4}^1\, x_{b+c+d+5}^0 \ldots x_n^0) \\
+ (-1)^{a+c+d-2}.\,(a+c+d+3)^{3/-1}.\,\mathfrak{C}(x_1^{b+1}\, x_2^1 \ldots x_{a+c+d+4}^1\, x_{a+c+d+5}^0 \ldots x_n^0) \\
+ (-1)^{a+b+d-2}.\,(a+b+d+3)^{3/-1}.\,\mathfrak{C}(x_1^{c+1}\, x_2^1 \ldots x_{a+b+d+4}^1\, x_{a+b+d+5}^0 \ldots x_n^0) \\
+ (-1)^{a+b+c-2}.\,(a+b+c+3)^{3/-1}.\,\mathfrak{C}(x_1^{d+1}\, x_2^1 \ldots x_{a+b+c+4}^1\, x_{a+b+c+5}^0 \ldots x_n^0) \\
+ (-1)^{c+d-1}.\,(c+d+2)^{2/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^1 \ldots x_{c+d+4}^1\, x_{c+d+5}^0 \ldots x_n^0) \\
+ (-1)^{b+d-1}.\,(b+d+2)^{2/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^1 \ldots x_{b+d+4}^1\, x_{b+d+5}^0 \ldots x_n^0) \\
+ (-1)^{b+c-1}.\,(b+c+2)^{2/-1}.\,\mathfrak{C}(x_1^{a+1}\, x_2^{d+1}\, x_3^1 \ldots x_{b+c+4}^1\, x_{b+c+5}^0 \ldots x_n^0) \\
+ (-1)^{a+d-1}.\,(a+d+2)^{2/-1}.\,\mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^1 \ldots x_{a+d+4}^1\, x_{a+d+5}^0 \ldots x_n^0) \\
+ (-1)^{a+c-1}.\,(a+c+2)^{2/-1}.\,\mathfrak{C}(x_1^{b+1}\, x_2^{d+1}\, x_3^1 \ldots x_{a+c+4}^1\, x_{a+c+5}^0 \ldots x_n^0) \\
+ (-1)^{a+b-1}.\,(a+b+2)^{2/-1}.\,\mathfrak{C}(x_1^{c+1}\, x_2^{d+1}\, x_3^1 \ldots x_{c+d+4}^1\, x_{c+d+5}^0 \ldots x_n^0) \\
+ (-1)^{d}.\,(d+1).\,\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^1 \ldots x_{d+4}^1\, x_{d+5}^0 \ldots x_n^0) \\
+ (-1)^{c}.\,(c+1).\,\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{d+1}\, x_4^1 \ldots x_{c+4}^1\, x_{c+5}^0 \ldots x_n^0) \\
+ (-1)^{b}.\,(b+1).\,\mathfrak{C}(x_1^{a+1}\, x_2^{c+1}\, x_3^{d+1}\, x_4^1 \ldots x_{b+4}^1\, x_{b+5}^0 \ldots x_n^0) \\
+ (-1)^{a}.\,(a+1).\,\mathfrak{C}(x_1^{b+1}\, x_2^{c+1}\, x_3^{d+1}\, x_4^1 \ldots x_{a+4}^1\, x_{a+5}^0 \ldots x_n^0) \\
+ \Sigma_h (-1)^h.\, \Sigma_i (-1)^i.\, \Sigma_k (-1)^k.\, \Sigma_l (-1)^l.\, \mathfrak{C}(x_1^{a-h}\, x_2^{b-i}\, x_3^{c-k}\, x_4^{d-l}\, x_5^0 \ldots x_n^0)\, A_{h+i+k+l+4} \\
\quad h = 0, 1, 2, \ldots a-1;\; i = 0, 1, 2, \ldots b-1;\; k = 0, 1, 2, \ldots c-1;\; l = 0, 1, 2, \ldots d-1
\end{array}\right\} \quad (15)
$$

7.

Quibus igitur ex praecedentibus satis clare elucebit, quanam ratione non solum propositiones (4) — (7) adhibendae sint ad alias easque aptas relationes evolvendas, verum etiam omnes, quae earum agmen sequuntur. Quare, quum etiam relationum inde natarum indoles et lex progressionis haud difficulter perspici possit, ulteriori calculorum explicatione haud opus esse videtur.

Quapropter jam primum animadvertamus necesse est, contineri in relationibus (8) — (15) revera legem recurrendo functiones formandi: quarum utique ope functiones $\mathfrak{C}(\ldots.)$ majorum exponentium aptissime ad functiones $\mathfrak{C}(\ldots.)$ minorum exponentium easque simpliciores reducuntur. At ejus reductionis neutiquam adepti essemus leges, si tam arctos disquisitionis constituissemus fines, ut particulares tantummodo functiones

$$\mathfrak{C}(x_1^{a+1}\, x_2^0 \ldots\ldots\ldots\ldots x_n^0)$$
$$\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^0 \ldots\ldots\ldots x_n^0)$$
$$\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^0 \ldots x_n^0)$$
$$\ldots\ldots\ldots\ldots\ldots$$

nobis tractandae sint; sed generalioribus consideratis functionibus

$$\mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots\ldots\ldots x_p^1\, x_{p+1}^0 \ldots x_n^0)$$
$$\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

legem in utroque functionum genere adhibendam consecuti sumus.

Deinde vero facile perspectu est, singulos unius seriei terminos propositiones (8) (10) (12) (14) esse, neque huic seriei generalis deesse potest terminus, sive generale ejusmodi theorema, quod omnes, quas dixi, propositiones implicite contineat. Verum etiam propositiones (9) (11) (13) (15) singuli unius theorematum seriei sunt termini.

Unde haud parvum doctrinae de symmetris functionibus augmentum increscere mihi videtur. Namque una tantummodo omnium, quas supra dedi, propositionum eaque particularis (9) hucusque in lucem prolata est. Verum etiam *Waringii* quidem nonnullae sunt relationes, quibus tamen cum eaque illa (9) in significatione formationisque lege ne lenissimum quidem est vinculum. Quare totus subsidiorum, quorum usus ad evolvendas symmetras functiones adhuc patuit, apparatus haud melius quam diversissimorum fragmentorum collectio eaque minime apta aestimari potest.

8.

Attamen relationibus (8) — (15) exhibendis etiam vera eaque uberrima subsidia sunt acquisita, quibus adjuvantibus problema illud supra propositum jam solvi potest. Ergo cum singulas particularesque functiones

$$\mathfrak{C}(x_1^{a+1}\, x_2^0 \ldots\ldots\ldots\ldots x_n^0)$$
$$\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^0 \ldots\ldots\ldots x_n^0)$$
$$\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^{c+1}\, x_4^0 \ldots x_n^0)$$
$$\ldots\ldots\ldots\ldots\ldots$$

tum generaliores

$$\mathfrak{S}\,(x_1^{a+1}\ x_2^1 \ldots\ldots\ldots\ldots x_p^1\ x_{p+1}^o \ldots . x_n^o)$$
$$\mathfrak{S}\,(x_1^{a+1}\ x_2^{b+1}\ x_3^1 \ldots\ldots . x_p^1\ x_{p+1}^o \ldots . x_n^o)$$
$$\mathfrak{S}\,(x_1^{a+1}\ x_2^{b+1}\ x_3^{c+1}\ x_4^1 . x_p^1\ x_{p+1}^o \ldots . x_n^o)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

per quantitatum $A_1\ A_2\ A_3 \ldots .$ functiones exprime.

Sed magna hic occurrit difficultas, cum functionum ex elementis $A_1\ A_2 \ldots .$ formandarum, quarum ope functionum symmetrarum valores evolvendi sunt, et generales et particulares prorsus incognitae sint proprietates. Quum autem functiones, quae huic consilio inserviant, profecto existere oporteat, eas quoque inveniri posse nemo dubitet; quare id praecipue agendum esse videtur, ut omni studio omniumque rerum accurata consideratione disquisitio instituatur.

Itaque hujus libelli ratio cum arctiores ejus, quam quos velim, fines constituat, id maxime consilium meum tendit, ut viam, qua in difficili ea materia ad felicem perducere rem liceat eventum, demonstrem. Quare eas maxime ipsarum $A_1\ A_2\ A_3 \ldots .$ functiones, quibus adjuvantibus valores functionum

$$\mathfrak{S}\,(x_1^{a+1}\ x_2^1 \ldots . x_p^1\ x_{p+1}^o \ldots . x_n^o).$$
$$\mathfrak{S}\,(x_1^{a+1}\ x_2^o \ldots . x_n^o).$$

exprimi possunt, explicaturus sum.

Exordium autem ita fiat, ut relationum (3) et (9) ope varii valores, quos functio

$$\mathfrak{S}\,(x_1^{a+1}\ x_2^o \ldots . x_n^o).$$

pro diversis ipsius a valoribus habere potest, evolvantur idoneoque digerantur ordine. Facile enim perspectu est, esse

$$\left.\begin{array}{l}
\mathfrak{S}\,(x_1^1\ x_2^o \ldots x_n^o) = 1.\ A_1 \\
\mathfrak{S}\,(x_1^2\ x_2^o \ldots x_n^o) = 1.\ A_1\ A_1 - 2.\ A_2 \\
\mathfrak{S}\,(x_1^3\ x_2^o \ldots x_n^o) = 1.\ A_1\ A_1\ A_1 - \left|\begin{array}{l} 1.\ A_1\ A_2 + 3.\ A_3 \\ 2.\ A_2\ A_1 \end{array}\right. \\
\mathfrak{S}\,(x_1^4\ x_2^o \ldots x_n^o) = 1.\ A_1\ A_1\ A_1\ A_1 - \left|\begin{array}{l} 1.\ A_1\ A_1\ A_2 + \\ 1.\ A_1\ A_2\ A^1 \\ 2.\ A_2\ A_1\ A_1 \end{array}\right|\begin{array}{l} 1.\ A_1\ A_3 - 4.\ A_4 \\ 2.\ A_2\ A_2 \\ 3.\ A_3\ A_1 \end{array} \\
\mathfrak{S}\,(x_1^5\ x_2^o \ldots x_n^o) = 1.\ A_1 A_1 A_1 A_1 A_1 - \left|\begin{array}{l} 1.\ A_1 A_1 A_1 A_2 + \\ 1.\ A_1 A_1 A_2 A_1 \\ 1.\ A_1 A_2 A_1 A_1 \\ 2.\ A_2 A_1 A_1 A_1 \\ \\ \\ \end{array}\right|\begin{array}{l} 1.\ A_1 A_1 A_3 - \\ 1.\ A_1 A_2 A_2 \\ 1.\ A_1 A_3 A_1 \\ 2.\ A_2 A_1 A_2 \\ 2.\ A_2 A_2 A_1 \\ 3.\ A_3 A_1 A_1 \end{array}\left|\begin{array}{l} 1.\ A_1 A_4 + 5.\ A_5 \\ 2.\ A_2 A_3 \\ 3.\ A_3 A_2 \\ 4.\ A_4 A_1 \\ \\ \\ \end{array}\right.
\end{array}\right\} \quad (\omega)$$

Quos jam si adhibeas valores, relationis (8) ope etiam sequentes ipsius functionis

$$\mathfrak{S}\,(x_1^{a+1}\ x_2^1 \ldots . x_p^1\ x_{p+1}^o \ldots . x_n^o)$$

valores exhiberi possunt:

$$\mathfrak{C}(x_1^2\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) = 1.\, A_1\, A_p - (p+1).\, A_{p+1}$$

$$\mathfrak{C}(x_1^3\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) = 1.\, A_1\, A_1\, A_p - \left| \begin{array}{l} 1.\, A_1\, A_{p+1} \\ 2.\, A_2\, A_p \end{array} \right. + (p+2).\, A_{p+2}$$

$$\mathfrak{C}(x_1^4\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) = 1.A_1 A_1 A_1 A_p - \left| \begin{array}{l} 1.A_1 A_1 A_{p+1} \\ 1.A_1 A_2 A_p \\ 2.A_2 A_1 A_p \end{array} \right. + \left| \begin{array}{l} 1.A_1 A_{p+2} \\ 2.A_2 A_{p+1} \\ 3.A_3 A_p \end{array} \right. - (p+3).\, A_{p+3}$$

$$\mathfrak{C}(x_1^5\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) = 1.\, A_1 A_1 A_1 A_1 A_p - \left| \begin{array}{l} 1.\, A_1 A_1 A_1 A_{p+1} \\ 1.\, A_1 A_1 A_2 A_p \\ 1.\, A_1 A_2 A_1 A_p \\ 2.\, A_2 A_1 A_1 A_p \end{array} \right. + \left| \begin{array}{l} 1.\, A_1 A_1 A_{p+2} \\ 1.\, A_1 A_2 A_{p+1} \\ 1.\, A_1 A_3 A_p \\ 2.\, A_2 A_1 A_{p+1} \\ 2.\, A_2 A_2 A_p \\ 3.\, A_3 A_1 A_p \end{array} \right. - \left| \begin{array}{l} 1.\, A_1\, A_{p+3} \\ 2.\, A_2\, A_{p+2} \\ 3.\, A_3\, A_{p+1} \\ 4.\, A_4\, A_p \end{array} \right. + (p+4).\, A_{p+4} \qquad (\omega^1)$$

9.

Verum e propositionibus (ω) et (ω^1) colligere licet, ad valores functionum

$$\mathfrak{C}(x_1^{a+1}\, x_2^0 \ldots x_n^0) \quad \text{et} \quad \mathfrak{C}(x_1^{a+1}\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0)$$

apte exprimendos combinationibus ad definitas summas, quas vocant, opus esse, iisque ita ex elementis $A_1\, A_2 \ldots\ldots$ condendis, ut in singulis numeri ipsis A annexi certam summam compleant. Quae quidem res, quatenus formandarum harum combinationum ratio jam nota est, perfecta esse videtur. Sed accuratius productorum conditione in relationibus (ω^1) contentorum perpensa, nova prorsusque, quod mihi videtur, incognita combinationum classis obveniet; quibus id quidem praecipue proprium est, quod ultimum combinationis elementum singulum tantummodo inter elementa $A_p\ A_{p+1}\ A_{p+2} \ldots\ldots$ locum habeat, dum in omnibus reliquis locis quodcunque ex elementis $A_1\, A_2 \ldots\ldots$ adesse potest.

Qui autem singulis productis sive combinationibus junguntur numeri, eos locales duntaxat elementorum, primum cujusque combinationis locum tenentium numeros esse, facile perspicies.

Hinc vero sanequam manifestum fit, novum adhuc et permagnum doctrinae de combinationibus campum tractandum restare, idque magis elucebit, si ad evolvendos valores earum, quae forma generali

$$\mathfrak{C}(x_1^{a+1}\, x_2^{b+1}\, x_3^0 \ldots\ldots x_n^0)$$

continentur, functionum relatione (11) utaris. Quare huic analyseos parti summo prae ceteris studio incumbendum est, ideoque amplificandae recteque explicandae hujus disciplinae in sequentibus initium faciam.

10.

Significet expressio

$$A\,(123\;.\;.\;.\;.)^{\binom{m+1}{s+1}} \qquad \text{(I)}$$

summam omnium, quas ex elementis $A_1\; A_2\; A_3\; .\;.\;.\;.$ ea quidem lege condere licet, combinationum, ut quaeque combinatio m + 1 ea contineat elementa, quorum localium sit s + 1 summa numerorum.

Ponatur deinde,

$$A\,[(123\,.\,.\,.)^{(m)};\;(z\;z+1\;z+2\,.\,.\,.)^{(1)}]_{z+s} \qquad \text{(II)}$$

esse signum summae combinationum ex elementis $A_1\; A_2\; A_3\; .\;.\;.\;.$ condendarum, quarum quidem ea sit conditio, ut

1) quaevis combinatio m + 1 elementa contineat;
2) in quaque combinatione localium summa numerorum sit $= z + s$;
3) quaevis combinatio per localem elementi, quod primum locum obtinet, numerum multiplicetur;
4) unum tantummodo ex elementis $A_z\; A_{z+1}\; A_{z+2}\; .\;.\;.\;.\;.$ combinationis locum ultimum sive $(m+1)^{tum}$ teneat;
5) quodque ex elementis $A_1\; A_2\; A_3\; .\;.\;.\;.\;.$ in combinationis loco 1^{mo}, 2^{do}, ... m^{to} esse possit.

11.

Jam vero secundum notandi legem in (II) positam expressio

$$A\,[(123\;.\;.\;.)^{(m)};\;(123\;.\;.\;.)^{(1)}]_{s+1} \qquad \text{(III)}$$

breviter indicat summam omnium ex elementis $A_1\; A_2\; A_3, .\;.\;.$ sub ea conditione formandarum combinationum, ut quaeque m + 1 ea, quorum sit s + 1 localium numerorum summa, elementa contineat, et ipsa per localem ejus elementi numerum multiplicetur, quod primum combinationis locum obtinet; et ut in ultimo combinationis pariter ac in alio quocunque loco quodque ex elementis $A_1\; A_2\; A_3\; .\;.\;.\;.$ esse possit.

Quae ergo combinationes, cum in formandis iis nulla elementorum a certis locis excludendorum habenda sit ratio, breviter ita habentur, ut primum omnes, quarum per signum

$$A\,(123\;.\;.\;.)^{\binom{m+1}{s+1}}$$

indicatur summa, combinationes condantur, et deinceps per locales elementorum, quae primum obtinent locum, numeros multiplicentur.

At vero accuratius eorum, qui singulis combinationibus, quae summam

$$A\,(123\;.\;.\;.\;.)^{\binom{m+1}{s+1}}$$

constituunt, junguntur, numerorum conditione perpensa, aliam eamque breviorem formandi rationem existere apparebit. Namque perspicies esse

$$A\,[(123\,.\,.\,.)^{(m)};\;(123\,.\,.\,.)^{(1)}]_{s+1} = \varphi.\;\;A\,(123\,.\,.\,.)^{\binom{m+1}{s+1}}$$

ubi φ certi est signum numeri, quem determinare licet.

Sint enim

$$V_1 \quad V_2 \quad V_3 \quad V_4 \ldots\ldots\ldots$$

combinationes diversae, quarum quaeque m + 1 elementa contineat ea, quorum locales summam s + 1 constituant numeri.

Deinde

$$V_1 \quad V_1^{I} \quad V_1^{II} \quad V_1^{III} \ldots\ldots\ldots$$

sint ejusmodi combinationes, quae mutata duntaxat eorundem elementorum serie inter se differant, ita, ut sit, si valores tantum respicias,

$$V_1 = V_1^{I} = V_1^{II} = V_1^{III} = \ldots\ldots\ldots$$

Eadem prorsus sit ratio ipsarum

$$V_2 \quad V_2^{I} \quad V_2^{II} \quad V_2^{III} \ldots\ldots\ldots$$

et

$$V_3 \quad V_3^{I} \quad V_3^{II} \quad V_3^{III} \ldots\ldots\ldots$$

etc.

Tum sint

$$\alpha_r \quad \alpha_r^{I} \quad \alpha_r^{II} \quad \alpha_r^{III} \ldots\ldots\ldots$$

locales elementorum numeri eorum, quae primos ipsarum V_r V_r^{I} V_r^{II} ..... locos obtinent.

Denique sit

$$v_r$$

numerus earum, quae ex ipsa V_r permutandis elementis combinationes eliciuntur.

Quibus positis erit

$$\begin{aligned} A\,(123\ldots\ldots)^{(m+1)}_{(s+1)} = {} & V_1 + V_1^{I} + V_1^{II} + V_1^{III} + \ldots\ldots \\ & + V_2 + V_2^{I} + V_2^{II} + V_2^{III} + \ldots\ldots \\ & + V_3 + V_3^{I} + V_3^{II} + V_3^{III} + \ldots\ldots \\ & + \ldots\ldots\ldots\ldots \end{aligned}$$

$$\begin{aligned} A\,[(123\ldots)^{(m)};\,(123\ldots)^{(1)}]_{s+1} = {} & \alpha_1.\ V_1 + \alpha_1^{I}.\ V_1^{I} + \alpha_1^{II}.\ V_1^{II} + \ldots\ldots \\ & + \alpha_2.\ V_2 + \alpha_2^{I}.\ V_2^{I} + \alpha_2^{II}.\ V_2^{II} + \ldots\ldots \\ & + \alpha_3.\ V_3 + \alpha_3^{I}.\ V_3^{I} + \alpha_3^{II}.\ V_3^{II} + \ldots\ldots \\ & + \ldots\ldots\ldots\ldots \end{aligned}$$

Est autem

$$\begin{aligned} V_1^{I} = V_1^{II} = V_1^{III} = \ldots\ldots\ldots = V_1 \\ V_2^{I} = V_2^{II} = V_2^{III} = \ldots\ldots\ldots = V_2 \\ V_3^{I} = V_3^{II} = V_3^{III} = \ldots\ldots\ldots = V_3 \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

et

$$\begin{aligned} V_1 + V_1^{I} + V_1^{II} + \ldots\ldots\ldots = v_1.\ V_1 \\ V_2 + V_2^{I} + V_2^{II} + \ldots\ldots\ldots = v_2.\ V_2 \\ V_3 + V_3^{I} + V_3^{II} + \ldots\ldots\ldots = v_3.\ V_3 \\ \ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}$$

---

hinc quoque

$$A\,(123\,\ldots\ldots)^{(m+1)}_{s+1} = v_1.\ V_1 + v_2.\ V_2 + v_3.\ V_3 + \ldots\ldots\ldots \qquad (\lambda)$$

et

$$\left.\begin{aligned} A\,[(123\ldots)^{(m)};\ (123\ldots)^{(1)}]_{s+1} = (\alpha_1 + \alpha_1{}^{1} + \alpha_1{}^{11} + \alpha_1{}^{111} + \ldots).\ V_1 \\ + (\alpha_2 + \alpha_2{}^{1} + \alpha_2{}^{11} + \alpha_2{}^{111} + \ldots).\ V_2 \\ + (\alpha_3 + \alpha_3{}^{1} + \alpha_3{}^{11} + \alpha_3{}^{111} + \ldots).\ V_3 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad (\lambda^1)$$

Jam animadvertere juvat, omne omnium, quibus ipsae $V_r\ V_r{}^1\ V_r{}^{11}\ldots$ constant, elementorum, toties primum obtinere locum, quoties secundum, tertium et quemvis alium. Unde colligere licet, eorum, quae primos ipsarum $V_r\ V_r{}^1\ V_r{}^{11}\ldots$ locos obtinent elementa, localium numerorum summam, sive summam $\alpha_r + \alpha_r{}^1 + \alpha_r{}^{11} + \alpha_r{}^{111} + \ldots.$ esse tantam, quantam eorum, quae secundos, tertios et alios quosve locos tenent.

Quodsi nunc ipsas $V_r\ V_r{}^1\ V_r{}^{11}\ldots..$ verticali dispositas esse ordine ponatur, verticales inde nascuntur $m + 1$ series, horizontales vero v_r. Eorum autem, quae verticalem quamcunque constituunt seriem elementa, localium summa numerorum est $= \alpha_r + \alpha_r{}^1 + \alpha_r{}^{11} + \ldots.$, eamque ob causam omnium, quotquot locales adsunt, numerorum summa est

$$= (m + 1).\ (\alpha_r + \alpha_r{}^1 + \alpha_r{}^{11} + \alpha_r{}^{111} + \ldots)$$

Eorum vero, quae horizontalem quamcunque constituunt seriem elementa, localium summa numerorum est $= s + 1$, eamque ob causam omnium, quotquot locales adsunt, numerorum summa est

$$= v_r.\ (s + 1)$$

Est ergo

$$(m + 1).\ (\alpha_r + \alpha_r{}^1 + \alpha_r{}^{11} + \ldots) = v_r.\ (s + 1)$$

hinc quoque

$$\alpha_r + \alpha_r{}^1 + \alpha_r{}^{11} + \ldots. = \frac{s+1}{m+1}.\ v_r.$$

Secundum hanc normam quodsi valores serierum

$$\begin{gathered} \alpha_1 + \alpha_1{}^1 + \alpha_1{}^{11} + \ldots.. \\ \alpha_2 + \alpha_2{}^1 + \alpha_2{}^{11} + \ldots.. \\ \alpha_3 + \alpha_3{}^1 + \alpha_3{}^{11} + \ldots.. \\ \ldots\ldots\ldots\ldots \end{gathered}$$

formantur, et in propositione (λ^1) substituuntur, prodit

$$A\,[(123\ldots)^{(m)};\ (123\ldots)^{(1)}]_{s+1} = \frac{s+1}{m+1}.\ (v_1.\ V_1 + v_2.\ V_2 + v_3.\ V_3 + \ldots)$$

Quam denique si cum relatione (λ) conjungis, oritur relatio

$$A\,[(123\ldots)^{(m)};\ (123\ldots)^{(1)}]_{s+1} = \frac{s+1}{m+1}.\ A\,(123\ldots.)^{(m+1)}_{s+1} \qquad (IV)$$

12.

Sed et eas nunc consideremus combinationes, quarum summa per signum

$$A\,[(123\ldots)^{(m)};\ (z\ z+1\ z+2\ldots)^{(1)}]_{z+s}$$

indicatur.

Quodsi enim ipsa A_{z+u} in ultimo plurium combinationum loco est posita, earum quidem summa per formam

$$W.\ A_{z+u} + W^{1}.\ A_{z+u} + W^{11}.\ A_{z+u} + \ldots\ldots$$

sive per productum

$$(W + W^{1} + W^{11} + \ldots\ldots).\ A_{z+u}$$

exprimi potest. Quantitatum $W\ W^{1}\ W^{11} \ldots\ldots$ autem quaeque est combinatio ex m elementis ejusmodi condita, quorum locales numeri summam $z + s - (z + u) = s - u$ constituunt, eaque ipsa per localem primo loco positi elementi numerum multiplicata; quare est

$$W + W^{1} + W^{11} + \ldots\ldots = A\,[(123\ldots)^{(m-1)};\ (123\ldots)^{(1)}]_{s-u}$$

Unde earum combinationum summa, in quibus ipsa A_{z+u} ultimum tenet locum, erit

$$= A\,[(123\ldots)^{(m-1)};\ (123\ldots)^{(1)}]_{s-u}.\ A_{z+u}$$

Quae autem ratiocinatio cum omnino vera sit, quicunque ipsius u sit valor, sive quodcunque ex elementis $A_z\ A_{z+1}\ A_{z+2} \ldots\ldots$ per A_{z+u} repraesentetur, sequitur ut veram quoque hanc ess oporteat relationem:

$$A\,[(123\ldots)^{(m)};\ (z\ z+1\ z+2\ldots)^{(1)}]_{z+s} = \Sigma_u\, A\,[(123\ldots)^{(m-1)};\ (123\ldots)^{(1)}]_{s-u}.\ A_{z+u} \quad (V)$$
$$u = 0,\ 1,\ 2,\ \ldots\ldots s - m.$$

Cujus quidem relationis veritas ab ipsius z valore non pendet; itaque casu particulari, quo $z = 1$, etiam est:

$$A\,[(123\ldots)^{(m)};\ (123\ldots)^{(1)}]_{s+1} = \Sigma_u\, A\,[(123\ldots)^{(m-1)};\ (123\ldots)^{(1)}]_{s-u}.\ A_{u+1} \quad (VI)$$
$$u = 0,\ 1,\ 2,\ \ldots\ s - m$$

13.

Quibus igitur expeditis ad functiones symmetras revertor.

Nam siquidem designandi rationem, quam in (II) et (III) posui, rite applices, teneasque in universum esse

$$A\,[(123\ldots)^{(0)};\ (z\ z+1\ z+2\ldots\ldots)^{(1)}]_{z+q} = (z+q).\ A_{z+q},$$

propositiones (ω) et (ω^1) in §. 8 breviter atque harmonice sequenti modo exhiberi possunt:

$$\left.\begin{aligned}
\mathfrak{S}(x_1^1\ x_2^0 \ldots\ldots x_n^0) &= A\,[(123\ldots)^{(0)};\ (123\ldots)^{(1)}]_1 \\
\mathfrak{S}(x_1^2\ x_2^0 \ldots\ldots x_n^0) &= A\,[(123\ldots)^{(1)};\ (123\ldots)^{(1)}]_2 \\
&\quad - A\,[(123\ldots)^{(0)};\ (123\ldots)^{(1)}]_2 \\
\mathfrak{S}(x_1^3\ x_2^0 \ldots\ldots x_n^0) &= A\,[(123\ldots)^{(2)};\ (123\ldots)^{(1)}]_3 \\
&\quad - A\,[(123\ldots)^{(1)};\ (123\ldots)^{(1)}]_3 \\
&\quad + A\,[(123\ldots)^{(0)};\ (123\ldots)^{(1)}]_3 \\
\mathfrak{S}(x_1^4\ x_2^0 \ldots\ldots x_n^0) &= A\,[(123\ldots)^{(3)};\ (123\ldots)^{(1)}]_4 \\
&\quad - A\,[(123\ldots)^{(2)};\ (123\ldots)^{(1)}]_4 \\
&\quad + A\,[(123\ldots)^{(1)};\ (123\ldots)^{(1)}]_4 \\
&\quad - A\,[(123\ldots)^{(0)};\ (123\ldots)^{(1)}]_4 \\
\mathfrak{S}(x_1^5\ x_2^0 \ldots\ldots x_n^0) &= A\,[(123\ldots)^{(4)};\ (123\ldots)^{(1)}]_5 \\
&\quad - A\,[(123\ldots)^{(3)};\ (123\ldots)^{(1)}]^5 \\
&\quad + A\,[(123\ldots)^{(2)};\ (123\ldots)^{(1)}]_5 \\
&\quad - A\,[(123\ldots)^{(1)};\ (123\ldots)^{(1)}]_5 \\
&\quad + A\,[(123\ldots)^{(0)};\ (123\ldots(^{(1)}]_5
\end{aligned}\right\}\ (\vartheta)$$

et

$$\left.\begin{aligned}
\mathfrak{S}(x_1^2\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) &= A[(123\ldots)^{(1)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+1} \\
&\quad - A[(123\ldots)^{(0)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+1} \\
\mathfrak{S}(x_1^3\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) &= A[(123\ldots)^{(2)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+2} \\
&\quad - A[(123\ldots)^{(1)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+2} \\
&\quad + A[(123\ldots)^{(0)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+2} \\
\mathfrak{S}(x_1^4\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) &= A[(123\ldots)^{(3)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+3} \\
&\quad - A[(123\ldots)^{(2)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+3} \\
&\quad + A[(123\ldots)^{(1)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+3} \\
&\quad - A[(123\ldots)^{(0)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+3} \\
\mathfrak{S}(x_1^5\, x_2^1 \ldots x_p^1\, x_{p+1}^0 \ldots x_n^0) &= A[(123\ldots)^{(4)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+4} \\
&\quad - A[(123\ldots)^{(3)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+4} \\
&\quad + A[(123\ldots)^{(2)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+4} \\
&\quad - A[(123\ldots)^{(1)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+4} \\
&\quad + A[(123\ldots)^{(0)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+4}
\end{aligned}\right\} \qquad (\vartheta^1)$$

14.

Sed quamquam relationes (ϑ) et (ϑ^1) legem certam eamque generalem satis luculenter indicare videntur, demonstrata tamen haec neutiquam aestimari potest. Quae ideo ut firma, qua opus est, demonstratione muniatur, primum propositionem

$$\mathfrak{S}(x_1^{g+1}\, x_2^0 \ldots x_n^0) = \Sigma_r\, (-1)^r.\ A[(123\ldots)^{(g-r)};\ (123\ldots)^{(1)}]_{g+1} \qquad (\vartheta_1)$$
$$r = 0,\ 1,\ 2,\ \ldots\ g$$

assumo veram esse particularibus iis casibus, quibus sit $g = 0,\ 1,\ 2,\ \ldots\ a-1$.

Cujus legis quidem vi in propositione (9) ponere licet

$$\mathfrak{S}(x_1^{a-h}\, x_2^0 \ldots x_n^0) = \Sigma_r\, (-1)^r.\ A[(123\ldots)^{(a-h-r-1)};\ (123\ldots)^{(1)}]_{a-h}$$
$$r = 0,\ 1,\ 2,\ \ldots\ a-h-1$$

Substitutione autem hac facta, evadit:

$$\mathfrak{S}(x_1^{a+1}\, x_2^0 \ldots x_n^0) = M + (-1)^a.\ (a+1).\ A_{a+1} \qquad \ldots(\Omega)$$

ubi brevitatis gratia posui

$$M = \Sigma_h\, (-1)^h.\ \Sigma_r\, (-1)^r.\ A[(123\ldots)^{(a-h-r-1)};\ (123\ldots)^{(1)}]_{a-h}.\ A_{h+1} \qquad (\eta)$$
$$r = 0,\ 1,\ 2,\ \ldots\ a-\alpha-1$$
$$h = 0,\ 1,\ 2,\ \ldots\ a-1$$

Jam ponatur

$$H^{(a-h-r-1)}_{a-h} = A[(123\ldots)^{(a-h-r-1)};\ (123\ldots)^{(1)}]_{a-h} \qquad (\eta^1)$$

ut fiat

$$M = \Sigma_h\, (-1)^h.\ \Sigma_r\, (-1)^r.\ H^{(a-h-r-1)}_{a-h}.\ A_{h+1}$$
$$r = 0,\ 1,\ 2,\ \ldots\ a-h-1$$
$$h = 0,\ 1,\ 2,\ \ldots\ a-1$$

ipsisque r et h indicati tribuantur valores, series inde natae sequenti possunt digeri ordine:

$$\begin{aligned} M = H_{a}^{(a-1)}.\ A_1 - H_{a}^{(a-2)}.\ A_1 + H_{a}^{(a-3)}.\ A_1 - \ldots + \ldots (-1)^{a-1}.\ H_{a}^{(0)}.\ A_1 \\ - H_{a-1}^{(a-2)}.\ A_2 + H_{a-1}^{(a-3)}.\ A_2 - \ldots + \ldots (-1)^{a-1}.\ H_{a-1}^{(0)}.\ A_2 \\ + H_{a-2}^{(a-3)}.\ A_3 - \ldots + \ldots (-1)^{a-1}.\ H_{a-2}^{(0)}.\ A_3 \\ \ldots\ldots\ldots\ldots \\ (-1)^{a-1}.\ H_{1}^{(0)}.\ A_a \end{aligned}$$

Unde primum colligitur, ipsam M verticalium a serierum haberi summam posse; quarum quidem cum $(w + 1)^{ta}$ sit

$$\begin{aligned} &= (-1)^w.\ \Sigma_t\ H_{a-t}^{(a-w-1)}.\ A_{t+1} \\ &= (-1)^w.\ \Sigma_t\ A[(123\ldots)^{(a-w-1)};\ (123\ldots)^{(1)}]_{a-t}.\ A_{t+1} \\ &\qquad t = 0, 1, 2, \ldots w \end{aligned}$$

fiat quoque necesse est

$$\begin{aligned} M = \Sigma_w\ (-1)^w.\ \Sigma_t\ A[(123\ldots)^{(a-w-1)};\ (123\ldots)^{(1)}]_{a-t}.\ A_{t+1} \\ t = 0, 1, 2, \ldots w \\ w = 0, 1, 2, \ldots a-1 \end{aligned}$$

Sedenim secundum propositionem (VI) est

$$\begin{aligned} \Sigma_t\ A[(123\ldots)^{(a-w-1)};\ (123\ldots)^{(1)}]_{a-t}.\ A_{t+1} = A[(123\ldots)^{(a-w)};\ (123\ldots)^{(1)}]_{a+1} \\ t = 0, 1, 2, \ldots w \end{aligned}$$

ergo etiam

$$\begin{aligned} M = \Sigma_w\ (-1)^w.\ A[(123\ldots)^{(a-w)};\ (123\ldots)^{(1)}]_{a+1} \\ w = 0, 1, 2, \ldots a-1 \end{aligned}$$

Quo quidem ipsius M valore in propositione (Ω) substituto, oritur relatio

$$\begin{aligned} \mathfrak{C}(x_1^{a+1}\ x_2^{0} \ldots x_n^{0}) = \Sigma_w\ (-1)^w.\ A[(123\ldots)^{(a-w)};\ (123\ldots)^{(1)}]_{a+1} + (-1)^a.\ (a+1).\ A_{a+1} \\ w = 0, 1, 2, \ldots a-1 \end{aligned}$$

unde, cum sit

$$(a + 1).\ A_{a+1} = A[(123\ldots)^{(0)};\ (123\ldots)^{(1)}]_{a+1}$$

concludere licet esse

$$\begin{aligned} \mathfrak{C}(x_1^{a+1}\ x_2^{0} \ldots x_n^{0}) = \Sigma_h\ (-1)^h.\ A[(123\ldots)^{(a-h)};\ (123\ldots)^{(1)}]_{a+1} \qquad (16) \\ h = 0, 1, 2, \ldots a \end{aligned}$$

Propositio autem haec ipsa et inde nascitur, si in (ϑ_1) ponatur g = a. Quodsi ergo propositio (ϑ_1) iis casibus sit vera, quibus g = 0, 1, 2, ... a — 1, eo quoque casu, quo g = a, hanc veram esse oportet. Enimvero ex iis, quae in (ϑ) sunt annotata, non potest non colligi, veram esse propositionem (ϑ_1) casibus iis, quibus g = 0, 1, 2, 3, 4; quare ea quoque valet pro g = 5, et per consequens pro g = 6, etc.

Hinc propositionem (16) sine ulla exceptione pro a = 0, 1, 2, ... veram esse concludere licet.

15.

Secundum legem (16) autem in propositione (8) ponere licet

$$\mathfrak{C}(x_1^{a-h}\, x_2^{o} \ldots x_n^{o}) = \Sigma_q\, (-1)^q.\ A\,[(123\ldots)^{(a-h-q-1)};\ (123\ldots)^{(1)}]_{a-h}$$
$$q = 0, 1, 2, \ldots a-h-1$$

unde prodit relatio

$$\mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_p^{1}\, x_{p+1}^{o} \ldots x_n^{o}) = M_1 + (-1)^a.\ (p+a).\ A_{p+a} \qquad \ldots\ldots (\Omega^1)$$

ubi brevitatis causa posui

$$M_1 = \Sigma_h\, (-1)^h.\ \Sigma_q\, (-1)^q.\ A\,[(123\ldots)^{(a-h-q-1)};\ (123\ldots)^{(1)}]_{a-h}.\ A_{p+h} \qquad (\eta_1)$$
$$q = 0, 1, 2, \ldots a-h-1$$
$$h = 0, 1, 2, \ldots a-1.$$

Adhibitis autem iis, quae in (η^1) posita sunt, fit

$$M_1 = \Sigma_h\, (-1)^h.\ \Sigma_q\, (-1)^q.\ H_{a-h}^{(a-h-q-1)}.\ A_{p+h}$$
$$q = 0, 1, 2, \ldots a-h-1$$
$$h = 0, 1, 2, \ldots a-1$$

et si ipsis q et h indicati tribuantur valores:

$$\begin{aligned} M_1 = H_a^{(a-1)}.\ A_p - H_a^{(a-2)}.\ A_p &+ H_a^{(a-3)}.\ A_p - \ldots + \ldots (-1)^{a-1}.\ H_a^{(o)}.\ A_p \\ - H_{a-1}^{(a-2)}.\ A_{p+1} &+ H_{a-1}^{(a-3)}.\ A_{p+1} - \ldots + \ldots (-1)^{a-1}.\ H_{a-1}^{(o)}.\ A_{p+1} \\ &+ H_{a-2}^{(a-3)}.\ A_{p+2} - \ldots + \ldots (-1)^{a-1}.\ H_{a-2}^{(o)}.\ A_{p+2} \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &\qquad (-1)^{a-1}.\ H_1^{(o)}.\ A_{p+a-1} \end{aligned}$$

Qua quidem serierum dispositione concessa elucebit, ipsam M_1 verticalium quoque a serierum summam haberi posse; quarum cum $(r+1)^{ta}$ sit

$$= (-1)^r.\ \Sigma_t\, H_{a-t}^{(a-r-1)}.\ A_{p+t}$$
$$= (-1)^r.\ \Sigma_t\, A\,[(123\ldots)^{(a-r-1)};\ (123\ldots)^{(1)}]_{a-t}.\ A_{p+t}$$
$$t = 0, 1, 2, \ldots r$$

erit

$$M_1 = \Sigma_r\, (-1)^r.\ \Sigma_t\, A\,[(123\ldots)^{(a-r-1)};\ (123\ldots)^{(1)}]_{a-t}.\ A_{p+t}$$
$$t = 0, 1, 2, \ldots r$$
$$r = 0, 1, 2, \ldots a-1$$

Deinde secundum propositionem (V) est

$$\Sigma_t\, A\,[(123\ldots)^{(a-r-1)};\ (123\ldots)^{(1)}]_{a-t}.\ A_{p+t} = A\,[(123\ldots)^{(a-r)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+a}$$
$$t = 0, 1, 2, \ldots r$$

hinc etiam

$$M_1 = \Sigma_r\, (-1)^r.\ A\,[(123\ldots)^{(a-r)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+a}$$
$$r = 0, 1, 2, \ldots a-1$$

Quo ergo ipsius M_1 valore in (Ω^1) substituto fit

$$\mathfrak{C}(x_1^{a+1}\, x_2^{1} \ldots x_p^{1}\, x_{p+1}^{o} \ldots x_n^{o}) =$$
$$= \Sigma_r\, (-1)^r.\ A\,[(123\ldots)^{(a-r)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+a} + (-1)^a.\ (p+a).\ A_{p+a}$$
$$r = 0, 1, 2, \ldots a-1$$

et cum sit

$$(p+a).\ A_{p+a} = A\,[(123\ldots)^{(o)};\ (p\ p+1\ p+2\ldots)^{(1)}]_{p+a}$$

concludere inde licet esse

$$\mathfrak{C}\,(x_1^{a+1}\,x_2^1\,..\,x_p^1\,x_{p+1}^0\,..\,x_n^0) = \Sigma_h\,(-1)^h.\,A\,[(123..)^{a-h};\,(p\;p+1\;p+2..)^{(1)}]_{p+a} \quad (17)$$

$$h = 0,\,1,\,2,\,...\,a$$

ubi ipsi a quemcunque ex serie 0 1 2 3 4 .... sumtum valorem tribuere potes.

Verum eadem fere ratione valores quoque functionum

$$\mathfrak{C}\,(x_1^{a+1}\;\;x_2^{b+1}\;\;x_3^0\;....\;x_n^0)$$

et

$$\mathfrak{C}\,(x_1^{a+1}\;\;x_2^{b+1}\;\;x_3^1\;....\;x_p^1\;x_{p+1}^0\;...\;x_n^0)$$

adhibitis propositionibus (10) et (11) jam elici possunt. Sed tamen in his mox eas, quibus ad hanc rem explicandam uti oportet, ipsarum A_1 A_2 A_3 ..... functiones magis aliquanto complicatam legem sequi videbis. Quodsi vero ab ipso disquisitionis principio functiones illas generaliores ejusmodi esse debere teneas, a quibus ad particulariores, easque in §. 10—§. 12 explicatas recedere possis, magnum inde et haudquaquam inutile statuendae earum indolis ac naturae adjumentum habebis.